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Axially symmetric vibrations in poroelastic solid cylindrical panel resting on elastic foundation

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Abstract. The axially symmetric vibrations of an isotropic poroelastic cylindrical panel resting on elastic foundations are investigated in the framework of Biot's theory. The effects of the surrounding elastic medium are considered using the spring constant of the Wrinkler type and the shear constant of the Pasternak type. The frequency equation is obtained for both pervious and impervious surfaces. Non dimensional phase velocity is computed as a function of wavenumber. Numerical results are presented graphically.

1. Introduction

The study of elastic foundations in cylindrical structures has wide applications in the field of Engineering, Geology and Biomechanics. Cylindrical panel plays an important role in Aerospace as they are of high specific strength and high specific stiffness. Even in human body, a number of muscoskeletal models of knee joint and skin employ different forms of elastic Wrinkler foundations. Propagation of axially symmetric waves in finite elastic cylinder is studied in [1]. In the said paper, vibrations corresponding to the different propagation constants for the same frequency parameter are applied to satisfy the boundary conditions. In [2], authors studied wave propagation in a homogenous isotropic cylindrical panel embedded on an elastic medium. In the said paper, three dimensional wave propagation in cylindrical panel embedded in an elastic medium (Wrinkler model) is investigated in the context of linear theory of elasticity. Free vibrations of circular cylindrical shell on Wrinkler and Pasternak foundations is studied in [3]. In said paper, it is concluded that elastic foundation affects radial vibrations mode frequency while pertaining to torsional and longitudinal remains are unaffected. Employing Biot's theory [4], axially symmetric vibrations of a poroelastic composite solid cylinder is investigated in [5]. In the said paper, frequency equation is obtained for nonaxially and axially symmetric vibrations each for pervious and impervious surfaces, and the results are compared with that of rule of mixture (RoM). In [6] authors investigated vibration characteristics of fluid filled cylindrical shells based on elastic foundations. In the said paper, frequencies are strongly affected when a cylindrical shell is attached with elastic foundation. Axially symmetric vibrations of fluid filled poroelastic circular shells is studied in [7]. In [8], authors investigated axially symmetric vibrations of finite composite poroelastic cylinders. In the said paper, non-dimensional phase velocity for propagating modes is computed as a function of ratio of length of cylinders in the absence of dissipation. To the best of knowledge, axially symmetric vibrations in poroelastic solid cylindrical panel resting on elastic foundation are not yet investigated. Therefore, in this paper the same is investigated in the framework of Biot's theory. Frequency equations are obtained for

both pervious and impervious surfaces. Non-dimensional phase velocity against the wavenumber for different elastic foundation are computed for three solids namely sandstone saturated with kerosene, water, and bone.

This paper is organized as follows. In section 2, governing equations, and solution of the problem are given. In section 3, frequency equations are derived. Numerical results are done in section 4. Finally, conclusions is given in section 5.

2. Governing equations and solution of the problem

The equations of motion of a poroelastic solid [4] in presence of dissipation (b) which in terms of displacement vectors are

$$\begin{aligned} &N\nabla^2\vec{u} + (A+N)\nabla e + Q\epsilon = \tfrac{\partial^2}{\partial t^2}(\rho_{11}\vec{u} + \rho_{12}\vec{U}) + b\tfrac{\partial}{\partial t}(\vec{u} - \vec{U}), \\ &\nabla(Qe + R\epsilon) = \tfrac{\partial^2}{\partial t^2}(\rho_{12}\vec{u} + \rho_{12}\vec{U}) - b\tfrac{\partial}{\partial t}(\vec{u} - \vec{U}), \end{aligned} \tag{1}$$

where, ∇^2 is the Laplace operator, $\vec{u}(u, v, w)$ and $\vec{U}(U, V, W)$ are solid and fluid displacements, e and ϵ are the dilatations of solid and fluid respectively; the symbols A, N, Q, R are all poroelastic constants; ρ_{ij} are mass coefficients. The constitutive relations are

$$\begin{aligned} &\sigma_{ij} = 2Ne_{ij} + (Ae + Q\epsilon)\delta_{ij}, (i, j = 1, 2, 3), \\ &s = Qe + R\epsilon. \end{aligned} \tag{2}$$

In eq. (2), e_{ij}'s are strain displacements, σ_{ij}'s are solid stresses and fluid pressure s, δ_{ij} the well known Kroneckar delta function. For axially symmetric vibrations, the displacements of solid $\vec{u}(u, 0, w)$ and fluid $\vec{U}(U, 0, W)$ which in terms of potential functions $\phi's$ and $\psi's$ are given below by

$$\begin{aligned} &u = \tfrac{\partial\phi_1}{\partial r} - \tfrac{\partial\psi_1}{\partial z}, \qquad && w = \tfrac{\partial\phi_1}{\partial z} + \tfrac{\partial\psi_1}{\partial r} + \tfrac{\psi_1}{r}, \\ &U = \tfrac{\partial\phi_2}{\partial r} - \tfrac{\partial\psi_2}{\partial z}, \qquad && W = \tfrac{\partial\phi_2}{\partial z} + \tfrac{\partial\psi_2}{\partial r} + \tfrac{\psi_2}{r}. \end{aligned} \tag{3}$$

For free harmonic waves travelling in the z- direction, we take

$$\begin{aligned} &\phi_1 = F_1(r)coskze^{i\omega t}, \qquad && \phi_2 = F_2(r)coskze^{i\omega t}, \\ &\psi_1 = G_1(r)sinkze^{i\omega t}, \qquad && \psi_2 = G_2(r)sinkze^{i\omega t}. \end{aligned} \tag{4}$$

In eq.(4), k is the wavenumber. ω is the frequency of the wave, i is the complex unity. By substituting eq. (4) in eq. (3), we obtain solid displacements as follows.

$$\begin{aligned} &u = -(C_1pK_1(pr) + C_2qK_1(qr) + A_1kK_1(dr))coskze^{i\omega t}, \\ &w = -(C_1kK_0(pr) + C_2kK_0(qr) + A_1dK_0(dr))sinkze^{i\omega t}. \end{aligned} \tag{5}$$

In eq. (5) C_1, C_2, A_1 are all arbitrary constants, $K_n(x)$ is the modified Bessel functions of second kind of order n and $p, q, d = k(1-\xi_1^2)$, $\xi_i = \frac{\omega}{kV_i}$,$i = 1, 2, 3$. Here $V_i(i = 1, 2, 3)$ are the dilatational wave velocities of first and second kind and shear wave velocity, respectively. Making use of eq.

(2) and eq. (5), we obtain the relevant stresses and fluid pressure that are given below

$$\begin{aligned}
&\sigma_{rr} + s - Ku - G\Delta u = (A_{11}(r)C_1 + A_{12}(r)C_2 + A_{13}(r)A_1)coskze^{i\omega t},\\
&\sigma_{rz} = (A_{21}(r)C_1 + A_{22}(r)C_2 + A_{23}(r)A_1)sinkze^{i\omega t},\\
&s = (A_{31}(r)C_1 + A_{32}(r)C_2)coskze^{i\omega t},\\
&\tfrac{\partial s}{\partial r} = (B_{31}(r)C_1 + B_{32}(r)C_2)coskze^{i\omega t},
\end{aligned} \tag{6}$$

where,

$$\begin{aligned}
&A_{11} = \tfrac{2Np}{r}K_1(pr) + (2Np^2 + (P - 2N + Q\delta_1^2)(p^2 - k^2) + (Q + R\delta_1^2)(p^2 - k^2)K_0(pr) - \tfrac{2Np}{r}K_1(pr),\\
&A_{13} = \tfrac{2Np}{r}K_1(dr) + 2NkK_0(dr) - \tfrac{2Np}{r}K_1(dr),\\
&A_{21} = 2NkpK_1(pr),\\
&A_{23} = N(d^2 - k^2)K_1(dr),\\
&A_{31} = (Q + R\delta_1^2)(p^2 - k^2)K_0(pr),\\
&A_{33} = 0,\\
&\delta_1^2 = \tfrac{(PR-Q^2)V_1^{-2} - (Rm_{11} - Qm_{12})}{Rm_{12} - Qm_{22}},
\end{aligned} \tag{7}$$

δ_2^2=similar expression as δ_1^2 with V_1 replaced by V_2,A_{12}, A_{22}, A_{32}=similar expression as A_{11}, A_{21}, A_{31},with p and δ_1^2,replaced by q and δ_2^2,respectively, and $m_{11} = \rho_{11} - ib\omega^{-1}$, $m_{12} = \rho_{12} + ib\omega^{-1}$, $m_{22} = \rho_{22} - ib\omega^{-1}$.

3. Boundary conditions and frequency equation

The boundary condition for stress free surface at $r = a$ for the pervious surface is

$$\sigma_{rr} + s - Ku - G\triangle u = \sigma_{rz} = s = 0. \tag{8}$$

The boundary condition for stress free surface at $r = a$ for an impervious surface is

$$\sigma_{rr} + s - Ku - G\triangle u = \sigma_{rz} = \tfrac{\partial s}{\partial r} = 0. \tag{9}$$

In eq. (8)and (9), $\triangle = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$, K is the foundation modulus given by $K = \frac{E}{(1+\mu)a}$, where E is the Young's modulus, μ is the poisson's ratio [9]. If the shear modulus G goes to zero, the Pasternak foundation will reduce to Wrinkler foundation [10]. Eq. (8)and (9) gives a system of three homogenous equations in the three arbitrary constants C_1, C_2, A_1 each for a pervious surface and impervious surface. A nontrivial solution can be obtained if the determinant of the coefficient vanishes. Accordingly, the frequency equation in the case of pervious surface is

$$|a_{ij}| = 0, i, j = 1, 2, 3, \tag{10}$$

where $a_{ij'}s$ are same as $A_{ij'}s$ with r is replaced by a in the eq. (7). In the case of an impervious surface the frequency equation becomes

$$|b_{ij}| = 0, i, j = 1, 2, 3, \tag{11}$$

here,

$$b_{31} = (Q + R\delta_1^2)p(p^2 - k^2)K_1(pa),$$

$$b_{32} = (Q + R\delta_2^2)q(q^2 - k^2)K_1(qa), \tag{12}$$

$$b_{33} = 0,$$

where $b_{ij} = a_{ij} \qquad i, j = 1, 2, j = 1, 2, 3.$

4. Numerical results

For the sake of numerical work, the dissipative coefficient b is taken zero and hence we obtained only real phase velocity. To analyze the frequency equations of axially symmetric vibrations of poroelastic cylindrical panel, it is convenient to introduce the following non dimensional parameters

$$a_1 = \frac{P}{H}, \quad a_2 = \frac{Q}{H}, \quad a_3 = \frac{R}{H}, \quad a_4 = \frac{N}{H},$$

$$d_1 = \frac{\rho_{11}}{\rho}, \quad d_2 = \frac{\rho_{12}}{\rho}, \quad d_3 = \frac{\rho_{22}}{\rho},$$

$$\tilde{x} = (\frac{V_0}{V_1})^2, \quad \tilde{y} = (\frac{V_0}{V_2})^2, \quad \tilde{z} = (\frac{V_0}{V_3})^2, \tag{13}$$

$$\rho = \rho_{11} + 2\rho_{12} + \rho_{22}, \quad H = P + 2Q + R, \quad V_0^2 = \frac{H}{\rho},$$

$$m = \frac{c}{c_0}, \quad c = \frac{\omega}{k}, \quad c_0^2 = \frac{N}{\rho}.$$

In the eq.(13), c is the phase velocity, m is the non-dimensional phase velocity, ka is the non-dimensional wavenumber. Employing the non-dimensional quantities in the frequency equation, we obtain a implicit relation between non-dimensional phase velocity and non-dimensional wavenumber. For the numerical process, three types of poroelastic solids are considered and then discussed. Of three poroelastic solids, two are sandstone saturated with kerosene and water, respectively [11, 12] and the third one is bony element. The physical parameters of first two materials pertaining to eq. (13) are given in the Table 1. Further, the values of bone poroelastic parameters A, N, Q, R and its mass coefficients ρ_{ij} are computed following the paper [13].The values of Young's modulus and Poisson ratio are taken to be 3×10^6 and 0.28, respectively as suggested in [13]. Phase velocity is computed using the bisection method implemented in MATLAB and the results are depicted in the figures1-8. In all the cases, curves are periodic and coincide in nature. Figures 1-8 show the plots of non-dimensional phase velocity against the wavenumber pertaining to different elastic foundations 10, 20, 50, and 100 respectively. From figures 1,3,5,7, it is observed that phase velocity of Material-I values are greater than phase velocity of Material-II for both pervious and impervious surface. From figure 1, 3, it is clear that phase velocity of Material-I pervious values are greater than phase velocity of Material-I impervious surface. Also, it is observed that the phase velocity for both pervious and impervious surface of Material-II is almost steady and constant beyond the wavenumber 1. From figures 5 and 7 it is observed that the phase velocities of Material-II both pervious and impervious surface is almost constant beyond the wavenumber 5. Figure-2 show the plot of non-dimensional phase velocity against non-dimensional wavenumber in the case of bone. From this figure, it is clear that phase velocity is same for both pervious and impervious surface beyond the wavenumber 7. From figure-4, it is observed that phase velocity is constant for both pervious and impervious surface beyond the wavenumber 5. From figure 6 and 8, it is clear that phase velocity is same for both pervious and impervious surface beyond the wavenumber 3.

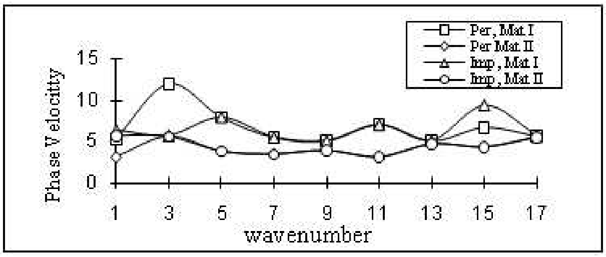

Figure 1. Variation of non-dimensional phase velocity with the wavenumber at elastic foundation ($K = 10$).

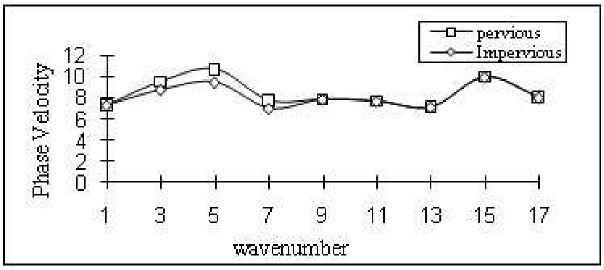

Figure 2. Variation of non-dimensional phase velocity with the wavenumber in the case of bone at elastic foundation ($K = 10$).

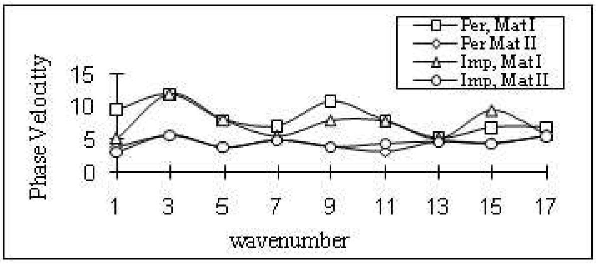

Figure 3. Variation of non-dimensional phase velocity with the wavenumber at elastic foundation ($K = 20$).

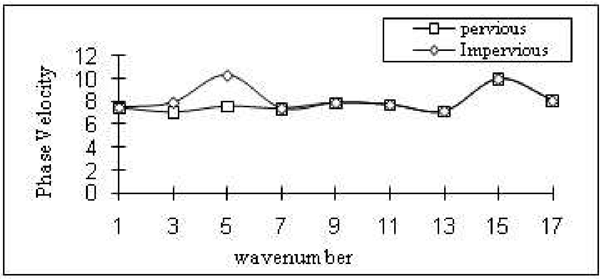

Figure 4. Variation of non-dimensional phase velocity with the wavenumber in the case of bone at elastic foundation ($K = 20$).

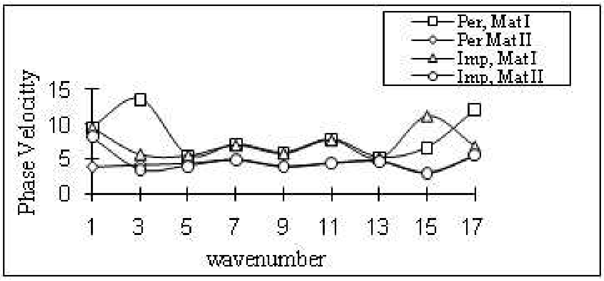

Figure 5. Variation of non-dimensional phase velocity with the wavenumber at elastic foundation ($K = 50$).

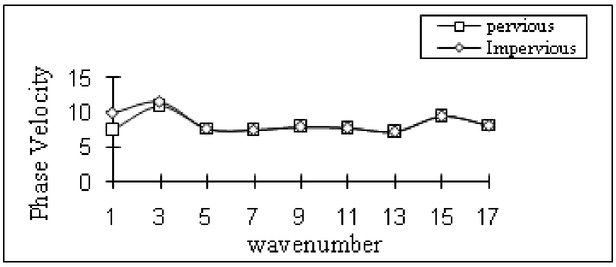

Figure 6. Variation of non-dimensional phase velocity with the wavenumber at elastic foundation ($K = 100$).

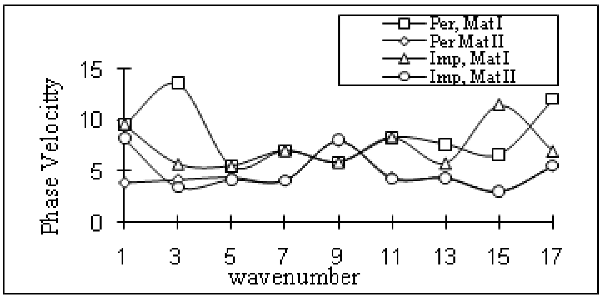

Figure 7. Variation of non-dimensional phase velocity with the wavenumber at elastic foundation ($K = 100$).

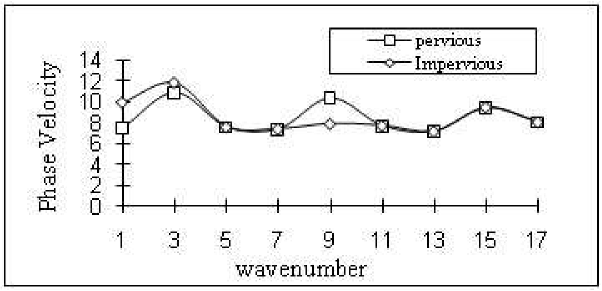

Figure 8. Variation of non-dimensional phase velocity with the wavenumber in the case of bone at elastic foundation ($K = 100$).

Table 1. Material parameters

Material parameters	Materia-I	Material-II
a_1	0.843	0.96
a_2	0.065	0.006
a_3	0.027	0.0289
a_4	0.234	0.412
d_1	0.901	0.876
d_2	-0.001	0
d_3	0.1	0.124
$\tilde{x}$	4.869	4.2977
$\tilde{y}$	0.998	0.912
$\tilde{z}$	3.85	2.126

5.Conclusion

Axially symmetric vibrations in poroelastic solid cylindrical panel resting on elastic foundation are investigated in the framework of Biot's theory. Non-dimensional phase velocity against non-dimensional wavenumber is computed for three types of poroelastic solids for different elastic foundations. Of three poroelastic solids, two are sandstone and third one is bony material. From the results, we can infer nature of surface has little influence over the values in the case of bone unlike sandstone cylinders.

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